Calculus Warm Up # 9-1

Fluid force on a vertical plane: F = (Pressure)(Area)

(weight)(depth)(length)(Δy)

w h(y) L(y)

$$F = w\int_c^d h(y)\,L(y)\,dy$$

(The object is living at depth [c, d])

Draw the situation twice and set up 2 different integrals to find the fluid force on a square object with 6 foot sides, centered 16 feet below the surface of the water.

1. x-axis in the center of the square.
2. x-axis at the bottom edge of the square.

HW Questions: Salmon WS

Calculus Mixed Review #3 Name_____ Team_____

1. $\int \frac{4x+4}{x^2+2x-4}dx$

$2\ln|x^2+2x-4|+C$

2. $\int \frac{x^2}{(x^3-1)^4}dx$

$-\frac{1}{9(x^3-1)^3}+C$

3. $\int \frac{1+e^{-x}}{e^{-x}}dx =$

$e^x + x + C$

4. $\int \frac{x^2+1}{x+1}dx =$

$\frac{x^2}{2} - x + 2\ln|x+1| + C$

5. $\int_{-1}^{0}\left(x^3+1\right)^3 dx =$ $\frac{81}{140}$

6. Use a middle Riemann sum with n = 4 and equal subintervals to approximate

$\int_0^4 x^3 dx \approx 62$

7. Use the trapezoidal rule with n = 4 to approximate.

$\int_0^1 \sqrt[3]{x}\,dx \approx 0.708$

8. Find the average value of on [0, 4].

$f(x) = x^3$

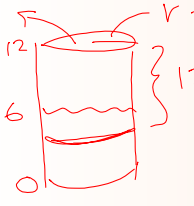
16

9. Find the work needed to stretch a spring 2 feet if it takes a force of 6 pounds to stretch it 8 inches.

$= 216$

10. A right cylinder with height 12 feet and diameter 4 feet is half filled with water. Find the work needed to empty it out the top.

13478.4π ft-lbs



$\Delta F = (62.4)\pi(2)^2 \Delta y$

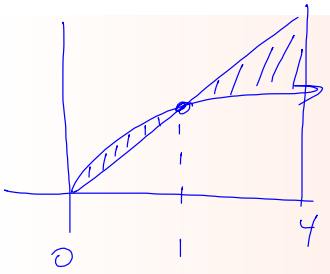
$W = 249.6\pi \int_0^6 (12-y)\,dy$

11. Find the fluid force on a vertical square object with sides of 3 feet that is centered 4 feet below the water surface.

$= 2246.4$ lbs

12. Sketch the graph and find the area between the functions on [0, 4].

$f(x) = \sqrt{x}$ $\quad f(x) = x$



$$A = \int_0^1 (\sqrt{x} - x)\,dx + \int_1^4 (x - \sqrt{x})\,dx$$

$$\left[\frac{2x^{3/2}}{3} - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - \frac{2x^{3/2}}{3}\right]_1^4$$

$$\frac{3}{}$$

13. Rotate the region from question 12 on [0,1] about the given line and find the volume of the solid created.

a) about y = 5 $\qquad$ b) about the y-axis

$\frac{3\pi}{2}$ $\qquad$ $\frac{2\pi}{15}$

8.4 Integrals of trigonometric functions

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$

$$\therefore \int \cos x\,dx = \sin x + C \qquad \int \sin x\,dx = -\cos x + C$$

Easy enough! Let's try a few...

$\int 6x^2 \sin x^3 \, dx$ Let $u = x^3$

$du = 3x^2 \, dx$

$= 2\int 3x^2 \sin x^3 \, dx$

$= 2\int \sin u \, du$

$= -2\cos u + C$

$= -2\cos x^3 + C$

$\int \sin^2(5x)\cos(5x)\,dx$ Let $u = \sin(5x)$

$du = 5\cos(5x)\,dx$

$= \frac{1}{5}\int [\sin(5x)]^2 \cdot \underbrace{5\cos(5x)\,dx}_{du}$

$[\sin(5x)]^2 \rightarrow u^2$

$= \frac{1}{5}\int u^2 \, du$

$= \frac{u^3}{15} + C$

$= \frac{\sin^3(5x)}{15} + C$

$$-\int \frac{-\sin x}{\cos^2 x}\,dx$$

let $u = \cos x$

$du = -\sin x\,dx$

$$\sec x + C$$

$$-\int \sin x \cos^3 x\,dx$$

let $u = \cos x$

$du = -\sin x\,dx$

$$-\int u^3\,du$$

$$-\frac{\cos^4 x}{4} + C$$

$\int \tan x \, dx \longrightarrow$ 😮 Uh Oh!

$\tan x$ is not anyone's basic derivative

Rewrite

$-\int \frac{-\sin x}{\cos x} \, dx$

let $u = \cos x$

$du = -\sin x \, dx$

$-\int \frac{1}{u} \, du$

$= -\ln|\cos x| + C$

HW: p. 456, # 1, 2, 7 - 10,

#27 - 37 odd, 45, 47

#35 → you need trig identities.

Individual Quiz Tuesday:

7.2, 7.6, 7.7